**1.3.1**. Extend equation (5) into a 4 by 4 factorization . What is the determinant of ?

**Sol**.

**1.3.2.** (1). Find the inverses of the 3 by 3 matrices and and in equation (5) .

(2). Write a formula for the th pivot of .

(3). Check that the , entry of is (on and below the diagonal) by multiplying or .

**Sol.** (1).

(2). is the formula for the th pivot of .

(3).

**1.3.3**. (1). Enter the matrix by the MATLAB command toeplitz([2,-1,0,0,0]).

(2). Compute the determinant and the inverse by det(K) and inv(K) . For a neater answer compute the determinant times the inverse.

(3). Find the , , factors of and verify that the entry of is .

**Sol**. (1).

>> K=toeplitz([2,-1,0,0,0])

K =

2 -1 0 0 0

-1 2 -1 0 0

0 -1 2 -1 0

0 0 -1 2 -1

0 0 0 -1 2

(2).

>> det(K)

ans =

6

>> inv(K)

ans =

0.8333 0.6667 0.5000 0.3333 0.1667

0.6667 1.3333 1.0000 0.6667 0.3333

0.5000 1.0000 1.5000 1.0000 0.5000

0.3333 0.6667 1.0000 1.3333 0.6667

0.1667 0.3333 0.5000 0.6667 0.8333

>> det(K)\*inv(K)

ans =

5.0000 4.0000 3.0000 2.0000 1.0000

4.0000 8.0000 6.0000 4.0000 2.0000

3.0000 6.0000 9.0000 6.0000 3.0000

2.0000 4.0000 6.0000 8.0000 4.0000

1.0000 2.0000 3.0000 4.0000 5.0000

(3).

>> [L,U]=lu(K)

L =

1.0000 0 0 0 0

-0.5000 1.0000 0 0 0

0 -0.6667 1.0000 0 0

0 0 -0.7500 1.0000 0

0 0 0 -0.8000 1.0000

U =

2.0000 -1.0000 0 0 0

0 1.5000 -1.0000 0 0

0 0 1.3333 -1.0000 0

0 0 0 1.2500 -1.0000

0 0 0 0 1.2000

>> D=diag(diag(U))

D =

2.0000 0 0 0 0

0 1.5000 0 0 0

0 0 1.3333 0 0

0 0 0 1.2500 0

0 0 0 0 1.2000

>> inv(L)

ans =

1.0000 0 0 0 0

0.5000 1.0000 0 0 0

0.3333 0.6667 1.0000 0 0

0.2500 0.5000 0.7500 1.0000 0

0.2000 0.4000 0.6000 0.8000 1.0000

**1.3.4**. The vector of pivots for is . This is d=(2:5)./(1:4), using MATLAB's counting vector i:j. The extra . makes the division act a component at a time. Find in the MATLAB expression for L=eye(4)-diag(,-1) and multiply L\*diag(d)\*L' to recover .

**Sol**.

>> D=(2:5)./(1:4); L=eye(4)-diag((1:3)./(2:4),-1)

L =

1 0 0 0

-1/2 1 0 0

0 -2/3 1 0

0 0 -3/4 1

>> K=L\*diag(D)\*L'

K =

2 -1 0 0

-1 2 -1 0

0 -1 2 -1

0 0 -1 2

**1.3.5**. If has pivots 2, 7, 6 with no row exchanges, what are the pivots for the upper left 2 by 2 submatrix (without row 3 and column 3)? Explain why.

**Sol**. where the upper-left 2 by 2 sub matrix goes . So 2 and 7 are its pivots.

**1.3.6**. How many entries can you choose freely in a 5 by 5 symmetric matrix? How many can you choose in a 5 by 5 diagonal matrixand lower triangular(with ones on its diagonal)?

**Sol**. entries can be chosen freely in.

entires can be chosen freely inand.

**1.3.7**. 7 Suppose is rectangular ( by ) and is symmetric ( by ).

1. Transpose to show its symmetry. What shape is this matrix?

2. Show why has no negative numbers on its diagonal.

**Sol**. 1. so it is symmetric. Its shapes is

2. so the diagonal is non-negative.

**1.3.8**. Factor these symmetric matrices into with the pivots in : and and

**Sol**.

**1.3.9**. The Cholesky command A=chol(K) produces an upper triangular with . The square roots of the pivots from are now included on the diagonal of (so Cholesky fails unless and the pivots are positive). Try the chol command on , , , and eps \* eye(3) .

**Sol**. >> K=toeplitz([2,-1,0]); T=K; T(1,1)=1; B=T; B(3,3)=1;

>> chol(K)

ans =

1.4142 -0.7071 0

0 1.2247 -0.8165

0 0 1.1547

>> chol(T)

ans =

1 -1 0

0 1 -1

0 0 1

>> chol(B)

Error using chol

Matrix must be positive definite.

>> chol(B+eps\*eye(3))

ans =

1.0000 -1.0000 0

0 1.0000 -1.0000

0 0 0.0000

**1.3.10**. The all-ones matrix ones(4) is positive semidefinite. Find all its pivots (zero not allowed). Find its determinant and try eig(ones(4)) . Factor it into a 4 by 1 matrix times a 1 by 4 matrix .

**Sol**. with pivot 1. .

>> eig(ones(4))

ans =

-0.0000

0

0.0000

4.0000

**1.3.11**. The matrix K=ones(4)+eye(4)/100 has all 1's off the diagonal, and 1.01 down the main diagonal. Is it positive definite ? Find the pivots by lu(K) and eigenvalues by eig(K) . Also find itsfactorization and inv(K) .

**Sol**. >> K=ones(4)+eye(4)/100

K =

1.0100 1.0000 1.0000 1.0000

1.0000 1.0100 1.0000 1.0000

1.0000 1.0000 1.0100 1.0000

1.0000 1.0000 1.0000 1.0100

>> eig(K)

ans =

0.0100

0.0100

0.0100

4.0100 % positive defiinte.

>> [L,U,P]=lu(K)

L =

1.0000 0 0 0

0.9901 1.0000 0 0

0.9901 0.4975 1.0000 0

0.9901 0.4975 0.3322 1.0000

U =

1.0100 1.0000 1.0000 1.0000

0 0.0199 0.0099 0.0099

0 0 0.0150 0.0050

0 0 0 0.0133 % pivots are on the diagonal

P =

1 0 0 0

0 1 0 0

0 0 1 0

0 0 0 1

% whereis the diagonal of

>> inv(K)

ans =

75.0623 -24.9377 -24.9377 -24.9377

-24.9377 75.0623 -24.9377 -24.9377

-24.9377 -24.9377 75.0623 -24.9377

-24.9377 -24.9377 -24.9377 75.0623

**1.3.12**. The matrix K=pascal(4) contains the numbers from the Pascal triangle (tilted to fit symmetrically into K) . Multiply its pivots to find its determinant. Factorintowhere the lower triangularalso contains the Pascal triangle!

**Sol**. >> K=pascal(4)

K =

1 1 1 1

1 2 3 4

1 3 6 10

1 4 10 20

>> [L,U,P]=lu(K)

L =

1.0000 0 0 0

1.0000 1.0000 0 0

1.0000 0.6667 1.0000 0

1.0000 0.3333 1.0000 1.0000

U =

1.0000 1.0000 1.0000 1.0000

0 3.0000 9.0000 19.0000

0 0 -1.0000 -3.6667

0 0 0 0.3333

P =

1 0 0 0

0 0 0 1

0 0 1 0

0 1 0 0

>> det(P')\*det(U)

ans =

1.0000

>> chol(K)'

ans =

1 0 0 0

1 1 0 0

1 2 1 0

1 3 3 1

**1.3.13**. The Fibonacci matrix is indefinite. Find its pivots. Factor it into . Multiply by this matrix 5 times, to see the first 6 Fibonacci numbers.

**Sol**. with pivotsand.

where

**1.3.14**. If , solve by hand the equation without ever finding itself. Solve and then (then is the desired equation ). is forward elimination and is back substitution:

**Sol**.

**1.3.15**. From the multiplication show that is the inverse of subtracts multiples of row 1 from lower rows. adds them back.

**Sol**.

**1.3.16**. Unlike the previous exercise, which eliminated only one column, show that

is not the inverse of

Write as to find the correct inverse (notice the order) :

and

**Sol**. . Sois not the inverse of.

**1.3.17**. By trial and error, find examples of 2 by 2 matrices such that

(1). (2). , with real entries in (3). , with no zeros in (4). , not allowing

**Sol**. (1). (2).

(3). (4).

**1.3.18**. Write down a 3 by 3 matrix with row 1 - 2 \* row 2 + row 3=0 and find a similar dependence of the columns – a combination of columns that gives zero.

**Sol**.

**1.3.19**. Draw these equations in their row form (two intersecting lines) and find the solution . Then draw their column form by adding two vectors: has column form .

**Sol**.

 

**1.3.20**. True or false: Every matrixcan be factored into a lower triangulartimes an upper triangular, with nonzero diagonals. Findandwhen possible: When is? ?

**Sol**. where nonzero diagonals happen if .

if